Abstracts

Ruscheweyh hyper-differential operator base of special monogenic polynomials

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Relying on the definition of differentiability of monogenic functions, the Ruscheweyh hyper-differential operator base of polynomials with values in a Clifford algebra is introduced. With this in hand the convergence problem of such Ruscheweyh hyper-differential operator base of polynomials is discussed.

By I. Cação (University of Aveiro, Portugal) M. Abul-Dahab (South Valley University, Egypt) M. Abul-Ez (Sohag University, Egypt).

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On the generalization of a Wold decomposition to a pair of isometries

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The classical Wold result gives a decomposition of an isometry into a unitary operator and a unilateral shift of certain multiplicity. As a consequence a geometrical model of an isometry is known. There is no simple generalization for a pair of isometries. By a canonical Wold decomposition of a pair of isometries we understand a decomposition into: a pair of unitary operators, a pair of unilateral shifts and two mixed pairs. A canonical Wold

decomposition of a pair of isometries holds with a strong doubly commutativity assumption. Moreover, for such a pair, a geometrical model is given. For only commuting pairs a functional model or Wold type decomposition (other than a canonical one) have been proposed by several authors.

The main result of the talk concerns compatible pairs of isometries; the much bigger class than doubly commuting pairs of isometries. A decomposition and a geometrical model of a compatible pair is fully described. General pairs of commuting isometries are also considered. A class of pairs of isometries having a canonical Wold decomposition other than doubly commuting pairs is given. The description of a unitary part in a completely non doubly commuting pair of isometries is presented.

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On extreme operators whose adjoints preserve extreme points

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Let X be a Banach spaces, which we call nice, such that any extreme operator T from a Banach space Y to X is a nice operator, that is, T^*, the adjoint of T, preserves extreme points. We get several necessary conditions for being nice. The main result is the characterization of nice finite-dimensional Banach spaces.

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Operator theory analysis of Dirichlet-Neumann boundary value problems in a half-plane with a crack

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The physical motivations behind the present study arise from problems of acoustic and electromagnetic time-harmonic plane wave diffraction. Thus, we will deal with boundary value problems for the Helmholtz equation. In particular, we prove the unique existence of solutions for different types of boundary value problems of wave diffraction by a half-plane with a crack. We will consider different combinations of Dirichlet and Neumann boundary conditions. Integral representations of the solutions are obtained upon the consideration of some associated operators. This is done in a Bessel potential spaces framework and for complex (non-real) wave numbers. The investigation is mostly based on the construction Wiener-Hopf and Hankel operators, and a factorization technique for a certain class of oscillating Fourier symbols which naturally arise from the problems. The talk is based on a joint work with D. Kapanadze.

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On the Essential ascent and descent for linear relations

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The concept of essential ascent and descent of operators is introduced and studied by several authors. In this work we extend this notion to the multivalued case, we investigate some properties and the related essential ascent and descent spectrums of a linear relation on a Banach space. We also focus on the stability of the essential ascent and descent spectrums under perturbations and we prove that the corresponding spectrums satisfy the

polynomial version and the fractional version of the Spectral Mapping Theorem.

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Cyclicity in reproducing kernel Hilbert spaces of analytic functions

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In this talk, we introduce a large family of reproducing kernel Hilbert spaces which include the classical Dirichlet spaces $\mathcal{D}_\alpha$. In this context, we completely characterize the cyclicity (with respect to the forward shift) of functions which are holomorphic on a neighborhood of the closed unit disc.

This is a joint work with Javad Mashreghi and Daniel Seco.

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The Linear Fractional Model Theorem and Alexandrov-Clark measures

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A remarkable result by Denjoy and Wolff states that every holomorphic self map φ of the open unit disc $\mathbb{D}$ of the complex plane, except the elliptic automorphisms, has an attractive fixed point to which the sequence of iterates $\{\varphi_n\}_{n\geq 1}$ converges uniformly on compacta: if there is not a fixed point in $\mathbb{D}$, then there is a unique boundary fixed point that does the job, called the *Denjoy-Wolff point*. The Denjoy-Wolff point provides a classification of the holomorphic self maps of $\mathbb{D}$ into four types: maps with interior

fixed point, hyperbolic maps, parabolic automorphism maps and parabolic non automorphism maps. We completely determine the convergence of the Alexandrov-Clark measures associated to maps falling in each group of such classification (joint work with Pekka Nieminen).

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Superposition operators on spaces of analytic functions

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If φ is an entire function then the superposition operator S_φ is defined by

$$S_\varphi(f)(z) = \varphi\left(f(z)\right), \quad f \in \mathcal{H}ol(\mathbb{D}), \quad z \in \mathbb{D}.$$

If X and Y are two subspaces of $\mathcal{H}ol(\mathbb{D})$, the question is: For which entire functions φ does the operator S_φ map X into Y?

This question has been studied for distinct pairs (X, Y) of spaces of analytic functions in the disc by distinct authors including Cámera, Giménez, Buckley, Fernández, Bonet, Vukotic and others.

In this talk we raise the question of characterizing the entire functions φ which transform the conformally invariant space Q^s $(0 \leq s < \infty)$ into the space of Dirichlet type $\mathcal{D}^p_\alpha$ $(0 < p < \infty,\ \alpha > -1)$ by superposition, and conversely. We shall pay a special attention to the case $s \geq 1$ and $\alpha = p-1$, that is, we shall deal mainly with the superposition operators between $BMOA$ or the Bloch space, $\mathcal{B}$, and the spaces $\mathcal{D}^p_{p-1}$ and compare them with those between $BMOA$ or $\mathcal{B}$ and the Hardy spaces H^p.

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On subordinated bounded holomorphic C_0-semigroups

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We give a positive answer to the the following question from [1, p. 63]: If $-A$ generates a (sectorially) bounded holomorphic C_0-semigroup on a Banach space X and ψ is a Bernstein function, then whether $-\psi(A)$ generates a bounded holomorphic C_0-semigroup on X too? Moreover, we show that ψ preserves the angle of holomorphicity of the semigroup. A partial positive answer to this question was given in [2, Proposition 7.4], where it was proved that $-\psi(A)$ generates a bounded holomorphic C_0-semigroup if $-A$ is the generator of a bounded holomorphic C_0-semigroup of angle greater than $\pi/4$.

This is joint work with Yu. Tomilov.

[1] A. Kishimoto and D. W. Robinson, *Subordinate semigroups and order properties,* J. Austral. Math. Soc. Ser. A **31** (1981), 59–76.

[2] C. Berg, K. Boyadzhiev and R. deLaubenfels, *Generation of generators of holomorphic semigroups,* J. Austral. Math. Soc. Ser. A **55** (1993), 246–269.

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Limit Operator Theory and Applications

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Let $p \in \{0\} \cup [1, \infty]$, $\mathbb{I} \in \{\mathbb{N}, \mathbb{Z}\}$ and X a complex Banach space. A bounded linear operator $A \in \mathcal{L}(\ell^p(\mathbb{I}, X))$ can be represented as a (bi-)infinite matrix acting on $\ell^p(\mathbb{I}, X)$. This viewpoint allows us to use matrix language, e.g. we can speak about *band operators* or *diagonals*. However, the infinite size of these matrices demands new concepts. How do matrix entries "at infinity" look like? The answer to this question leads to the notion of *limit operators* and *operator spectra*. It turns out that there is a close relationship

between the spectra of the limit operators and the essential spectrum of A. This connection has immediate consequences for various classes of operators.

We also want to talk about solving linear equations of the form $Ax = b$. A very natural approach in this matrix setting is the *Finite Section Method (FSM)*. Whether this method is applicable or not is highly dependent on the matrix A. With the help of limit operators a necessary and sufficient condition can be formulated.

We conclude with some results of ongoing work with Marko Lindner and Markus Seidel [1] that emphasize the algebraical duality between A and its limit operators.

[1] R. Hagger, M. Lindner, M. Seidel, *Essential pseudospectra and essential norms of band-dominated operators*, in preparation.

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Concrete examples of $\mathscr{H}(b)$ spaces

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In the late 1960s and early 1970s, de Branges and Rovnyak introduced and studied the class of spaces $\mathcal{H}(b)$ which are now also called de Branges-Rovnyak spaces; here b is a an analytic function on the unit disk bounded by 1 (or more generally a Schur class function). These spaces play an important role in operator theory since, via the compressed shift and the Sz.-Nagy-Foias theory, they serve as the model for a wide class of contraction operators on Hilbert spaces. $\mathcal{H}(b)$ spaces can be defined in different ways: as the reproducing kernel Hilbert space for a certain positive definite kernel associated with b, or as the range space of the defect operator for the Toeplitz operator $T_{\overline{b}}$. It is well known that when b is an inner function then $\mathcal{H}(b)$ is the classical model space $K_b = H^2 \ominus bH^2$. However, in the general case, the exact contents of $\mathcal{H}(b)$ is mysterious. What functions belong to $\mathcal{H}(b)$? The aim of this talk is to give an explicit description of de Branges-Rovnyak spaces $\mathcal{H}(b)$

when b is of the form q^r, where q is a rational outer function in the closed unit ball of H^∞ and r is a positive number.

This is joint work with Emmanuel Fricain (Lille) and William T. Ross (Richmond).

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On a Weighted Singular Integral Operator with Shifts and Slowly Oscillating Data

Alexei Karlovich
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Let α, β be orientation-preserving diffeomorphism (shifts) of $\mathbb{R}_+ = (0, \infty)$ onto itself with the only fixed points 0 and ∞ and U_α, U_β be the isometric shift operators on $L^p(\mathbb{R}_+)$ given by $U_\alpha f = (\alpha')^{1/p}(f \circ \alpha)$, $U_\beta f = (\beta')^{1/p}(f \circ \beta)$, and $P_2^\pm = (I \pm S_2)/2$ where

$$(S_2 f)(t) := \frac{1}{\pi i} \int_0^\infty \left(\frac{t}{\tau}\right)^{1/2-1/p} \frac{f(\tau)}{\tau - t}\, d\tau, \quad t \in \mathbb{R}_+,$$

is the weighted Cauchy singular integral operator. We prove that if α', β' and c, d are continuous on $\mathbb{R}_+$ and slowly oscillating at 0 and ∞, and

$$\limsup_{t \to s} |c(t)| < 1, \quad \limsup_{t \to s} |d(t)| < 1, \quad s \in \{0, \infty\},$$

then the operator $(I - cU_\alpha)P_2^+ + (I - dU_\beta)P_2^-$ is Fredholm on $L^p(\mathbb{R}_+)$ and its index is equal to zero. Moreover, its regularizers are described. The talk is based on a joint work with Yuri Karlovich and Amarino Lebre.

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On the spectra of quasianalytic contractions

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The hyperinvariant subspace problem for asymptotically non-vanishing contractions can be reduced to the quasianalytic case. Every quasianalytic Hilbert space contraction T is of class C_{10}, that is $\lim_n \|T^{*n}h\| = 0 < \lim_n \|T^n h\|$ holds for all non-zero vectors h. Complete characterization of possible spectra of C_{10}-contractions is known, see Section IX.2 in [NFBK]. We study the problem, whether quasianalycity imposes more restrictive spectral behaviour. Partial answers are given for natural questions arisen here.

The talk is based on a joint work with A. Szalai, accepted for publication in the *Proc. Amer. Math. Soc.*.

[NFBK] B. Sz.-Nagy, C. Foias, H. Bercovici, L. Kérchy, *Harmonic Analysis of Operators on Hilbert Space*, Revised and Enlarged Edition, Springer, 2010.

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Constructive approximate Wiener-Hopf factorisation with application to Daniele-Khrapkov matrices

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This talk will be on the stability of scalar and matrix Wiener-Hopf factorisation. Wiener-Hopf factorisation is the splitting of a given matrix function on a real line into the product of a function analytic in the upper half-plane, a function analytic in the lower half-plane and the third term containing the partial indices. Such factorisation has many important applications for example wave propagation affected by a sudden change of geometry. The stability of factorisation is important for two reasons: computational and

physical. Computational stability is vital in various algorithmic implementation of the factorisation. Physical models, which use Wiener-Hopf equation, require stability since real life measurements can only be taken with a certain precision. After reviewing scalar Wiener-Hopf factorisation and rational approximation results about the stability of the Daniele-Khrapkov matrices will be presented.

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Wandering vectors and related decomposition of isometries

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We show that for any isometry V acting on a Hilbert space H there is a unique decomposition:

$$H = H_0 \oplus H_w,$$

hyperreducing V and such, that

- H_w is a span of all wandering vectors,
- the isometry V restricted to H_0 is unitary and its every invariant subspace is reducing.

The above decomposition strongly corresponds with Lebesgue decomposition but is essentially different from it.

The coauthors of the presented result are Zbigniew Burdak, Patryk Pagacz and Marek Słociński.

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The magnetic Laplacian in shrinking tubular neighbourhoods of hypersurfaces

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The Dirichlet Laplacian between two parallel hypersurfaces in Euclidean spaces of any dimension in the presence of a magnetic field is considered in the limit when the distance between the hypersurfaces tends to zero. We show that the Laplacian converges in a norm-resolvent sense to a Schroedinger operator on the limiting hypersurface whose electromagnetic potential is expressed in terms of principal curvatures and the projection of the ambient vector potential to the hypersurface. As an application, we obtain an effective approximation of bound-state energies and eigenfunctions in thin quantum layers. This is joint work with Nicolas Raymond and Matej Tusek.

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Estimates for the dimension of the kernel of a singular integral operator with shift

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Some estimates for the dimension of the kernel of the singular integral operator $I - cUP_+$: $L_2^n(\mathbb{T}) \to L_2^n(\mathbb{T})$, with a non-Carleman shift are obtained, where P_+ is the Cauchy projector, U is an isometric shift operator and $c(t)$ is a continuous matrix function. It is supposed that the shift has a finite set of fixed points and all the eigenvalues of the matrix $c(t)$ at the fixed points, simultaneously belong either to the interior of the unit circle $\mathbb{T}$ or to its exterior. Some of the obtained results can be used in the case of an operator with a general shift.

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The use of sheaves in operator theory on C^*-algebras

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We will first briefly review the sheaf theory for C^*-algebras that was developed together with Pere Ara, Barcelona. We shall then apply this to obtain information on derivations on C^*-algebras.

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Coherent state transforms on compact Lie groups, complex time evolution and quantization

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Segal-Bargmann unitary transforms for a compact Lie group K are unitary maps from L^2 spaces of functions on K to spaces of holomorphic L^2 functions on the complexification $K_{\mathbb{C}}$ of K. We will describe the relation of these transforms with complex time evolution in geometric quantization. The relation with non Hermitian Quantum Mechanics will also be discussed.

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Two algorithms for constructing the numerical range of a banded biperiodic Toeplitz operator in a Krein space

Ana Cristina Becerra Nata
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In 1972, Klein prove that the numerical range of a Toeplitz operator, T_f, in a Hilbert space equals $\operatorname{conv}(f(\mathbb{D}))$, where f is the symbol of the operator, and $\mathbb{D}$ is the complex unit circle. In this talk, we extend this result

considering the characterization of the numerical range of a banded biperiodic Toeplitz operator defined in a Krein space. We prove that in this case, the Krein space numerical range coincides with the boundary of the pseudo-convex hull of a family of hyperbolical discs. Furthermore, the parametric equations of the boundary generating curves of $W_J(T_f)$ are deduced. As a consequence, we provide two algorithms that accurately exhibit the boundary of $W_J(T_f)$.

This talk is based on a joint work with Professors Natália Bebiano and J. P. da Providência from the University of Coimbra, Portugal.

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Boundedness of weighted composition operators on spaces of holomorphic functions

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We explain how the boundedness of a weighted composition operator on the Hardy-Hilbert space on the disc or half-plane implies its boundedness on a class of related spaces, including weighted Bergman spaces. This generalizes some known results for Toeplitz operators and composition operators. The methods used involve the study of lower-triangular and causal operators.

This is joint work with Isabelle Chalendar (Lyon).

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C-symmetric operators and its preanihilator

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Let $\mathcal{H}$ be a complex separable Hilbert space. Let C be an isometric antilinear involution in $\mathcal{H}$. A bounded operator $T \in B(\mathcal{H})$ is called *C-symmetric*, if $CTC = T^*$. Let $\mathcal{C}$ denote the set of all C-symmetric operators.

C–symmetric operators and the whole set $\mathcal{C}$ was intensively studied recently. There were many examples of C–symmetric operators such as Jordan blocks, truncated Toeplitz operators, Hankel operators ect. The aim of the talk is to present the description of the preanihilator of the space of all C–symmetric operators. It will be shown that the subspace of all C-symmetric operators is transitive and 2-reflexive or even 2-hyperreflexive. It means that the preanihilator of $\mathcal{C}$ does not contain any rank-one operators and rank-two operators are dense in the preanihilator. Moreover, the description all rank-two operators in this preanihilator is given.

Joint work with K. Kliś–Garlicka.

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Spectra of two-dimensional Dirac operator with zigzag boundary conditions

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We study the spectral properties of the two-dimensional Dirac operator on bounded domains together with the appropriate boundary conditions. This boundary value problem corresponds to a (continuous) model for graphene nanoribbons. We show the local compactness of resolvent, the existence of a sequence of finite multiplicity eigenvalues converging to zero, and analyze the effect of potential perturbations on the localization of essential spectrum.

The results were obtained in collaboration with P. Freitas (GFM, University of Lisbon).

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On some (relatively) recent results concerning the numerical range

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The field of values (also known as the numerical range) of a given matrix A is by definition the range of the function $f : x \to (Ax, x)$ considered on the unit sphere. We will discuss the properties of its inverse, in particular, the extent to which the latter is multivalued, and the sense in which it may (or may not) be continuous.

Time permitting, we will also give a precise description of the sets which can be realized as the numerical range of some matrices.

The results are mostly from the joint work with B. Lins, T. Leake, and J.W. Helton.

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Mixed BMO and boundedness of Hankel operators

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The classical Hankel operator H_ϕ is defined on the Hardy space H^2 by $H_\phi(f) = P^{H^\perp}(\phi f)$. A function u on the circle T is said to be of 'bounded mean oscillation' if , for u_I equal to the average of the function u on the interval I on the circle, the supremum of the values of $\frac{1}{|I|}\int_I |u(y) - u_I|dy$ is bounded. It is well known that a function ϕ on the circle has bounded mean oscillation if and only if the Hankel operators H_ϕ and $H_{\overline{\phi}}$ are bounded. This one variable result becomes more complicated in several variables as there are many ways to define Hankel operators and also multiple definitions of 'BMO' functions. Work on the several variable case was initiated by Cotlar and Sadosky in the 1990's , continued in the 21st century by Sadosky-Ferguson,

Ferguson-Lacey and others. I would like to talk about recent work on the subject with Petermichl, Pipher, and Ou.

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Schur coupling and other equivalence relations for operators on a Hilbert space

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For operators on Hilbert spaces, we show that several equivalence relations actually coincide. In particular, Schur coupling coincides with matricial coupling. We also provide a concrete description of this equivalence relation in several cases, for instance for compact operators. The analogue question for general Banach spaces remains open.

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Hankel and Toeplitz operators on generalized Fock spaces

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We discuss some recent results on boundedness, compactness and Schatten class membership of Toeplitz operators acting on Fock spaces. The case of standard weighted Fock spaces is well understood, and closely related to the theory of these operators on Bergman spaces.

In this talk we focus on the case of generalized Fock spaces. A characterization of bounded and compact Toeplitz operators was obtained only very recently by Schuster and Varolin. Our work is concerned with further

properties of Toeplitz and Hankel operators on these spaces. In particular, we give a characterization of Schatten class Toeplitz operators with positive measure symbols in terms of the Berezin transform and also in terms of the standard averaging function of the symbol.

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